

# **Kurt Godel's Ontological Proof Of The Existence Of God**

## **Hebrews 11:6 NIV**

**<sup>6</sup> And without faith it is impossible to please God, because anyone who comes to him must believe that he exists and that he rewards those who earnestly seek him.**

As Heb 11:6 says without Faith we can not please God. Thus, the foundation of our Christian walk is Faith in Jesus. The following "Proof" is just a way of showing how one of the greatest mathematical minds of the 20<sup>th</sup> century was a Believer. His wife said while Kurt Godel did not attend church, he believed and read the Bible on a regular basis. Some have said that Godel did not publish the proof while he was alive to avoid ridicule from other scientist.

**Here is a line-by-line explanation of his proof by**

**ROBERT J. MARKS AND SAMUEL HAUG**

**FROM THE MIND MATTERS WEBSITE**

Kurt Gödel, an intellectual giant of the 20th century, offered a mathematical proof that God exists. Those who suffer from math anxiety admire what the theorem (shown below) claims to do, but have absolutely no idea what it means. Our goal is to explain, in English, what Gödel's existence of God proof says.

- Ax. 1.  $(P(\varphi) \wedge \Box \forall x(\varphi(x) \Rightarrow \psi(x))) \Rightarrow P(\psi)$
- Ax. 2.  $P(\neg\varphi) \Leftrightarrow \neg P(\varphi)$
- Th. 1.  $P(\varphi) \Rightarrow \Diamond \exists x \varphi(x)$
- Df. 1.  $G(x) \Leftrightarrow \forall \varphi(P(\varphi) \Rightarrow \varphi(x))$
- Ax. 3.  $P(G)$
- Th. 2.  $\Diamond \exists x G(x)$
- Df. 2.  $\varphi \text{ ess } x \Leftrightarrow \varphi(x) \wedge \forall \psi(\psi(x) \Rightarrow \Box \forall y(\varphi(y) \Rightarrow \psi(y)))$
- Ax. 4.  $P(\varphi) \Rightarrow \Box P(\varphi)$
- Th. 3.  $G(x) \Rightarrow G \text{ ess } x$
- Df. 3.  $E(x) \Leftrightarrow \forall \varphi(\varphi \text{ ess } x \Rightarrow \Box \exists y \varphi(y))$
- Ax. 5.  $P(E)$
- Th. 4.  $\Box \exists x G(x)$

Gödel's proof shows the existence of God is a necessary truth. The idea behind the truth is not new and dates back to Saint Anselm of Canterbury (1033-1109). Great scientists and philosophers, including Descartes and Leibniz, have reconsidered and refined Anselm's argument. Gödel appears to be the first, however, to present the argument using mathematical logic.



Kurt Gödel

In any development of a mathematical theory, there are foundational axioms on which the theory is built. An axiom is assumed to be a fundamental truth to be accepted as obvious. Gödel's four axioms in his proof are labeled **Ax**.

The second elements of the proof are the definitions, denoted by **Df**. To talk about a topic, objects in the discussion need to first be defined.

The final elements of a proof are the theorems, denoted by **Th**. These are the claims proven throughout the course of the proof built on axioms and definitions. Intermediate theorems can also be used to help prove later theorems.

In Gödel's proof, lower case letters in English, like  $x$  and  $y$ , denote objects. Greek letters denote properties. Objects have properties. A cucumber is an object. Green is one of

its properties. In probability, the null set has the property its probability is zero.

There are also the strange looking characters in Gödel's proof. Most are simply shorthand notation for phrases often used by mathematicians. For example, the upside down A is read as "for every" and the backwards E means "there exists." Rather than go through each symbol, we have chosen to give a direct English translation of each math line.

### Line by Line Explanation

We will now analyze each line of Gödel's ontological proof of the existence of God. After each line of Gödel's proof, we give an English translation that a mathematics professor might vocalize out loud to a classroom full of students. These direct translations can be as opaque as the underlying mathematics. Below each translation is elaboration in plain English of the meaning of the math sometimes illustrated by examples.

The first line in Gödel's proof is an axiom:

$$\text{Ax. 1. } (P(\varphi) \wedge \Box \forall x(\varphi(x) \Rightarrow \psi(x))) \Rightarrow P(\psi)$$

Translation: “Axiom 1. If  $\phi$  is a positive property and it is necessary that for every  $x$  if  $x$  has the property  $\phi$  then  $x$  has the property  $\psi$ , then  $\psi$  is a positive property.”

This first line discusses positive properties. It says if a positive property always causes another property, that second property must also be positive. When Gödel discusses these “positive” properties, he is talking about it in the sense of a “good” property. For example, consider the positive property of tasting good. When food tastes good, it is enjoyable to eat. By this axiom, the property of being enjoyable to eat would necessarily be a positive property also because it is always caused by food tasting good, which is a positive property.

For an example more relevant to the aim of the proof, consider an attribute of God, such as being loving. Being loving is a positive property, and if something is loving it is also patient and kind (1 Corinthians 13:4). By this axiom, being patient and kind is also a positive property because it is always caused by the positive property of being loving.

$$\text{Ax. 2. } P(\neg\varphi) \Leftrightarrow \neg P(\varphi)$$

Translation: “Axiom 2. Not-phi is a positive property if and only if phi is not a positive property.”

Here, Gödel further discusses positive properties. The line says that lacking a positive property is the same thing as having a negative property. A property has to be either good or not good. There is no in-between. As an example, lacking the cancer-free property is the same thing as having cancer.

**Th. 1.**  $P(\varphi) \Rightarrow \Diamond \exists x \varphi(x)$

Translation: “Theorem 1. If property phi is a positive property then it is possible that there exists an x such that x has property phi.”

This line is the first proved item in the proof. It reads: If a property is positive, then it is possible that an object has that property. It is possible that good things exist.

**Df. 1.**  $G(x) \Leftrightarrow \forall \varphi (P(\varphi) \Rightarrow \varphi(x))$

Translation: “Definition 1. Object x has the godlike property if and only if for every property phi, if phi is a positive property, then x has property phi.”

This line defines “God” in the context of Gödel’s proof. Gödel does this by defining a “godlike” object. In order for an object to be godlike, it must have every good property. From Axiom 2 we know that this also means

that this godlike object can have no negative properties (e.g. if God has the cancer-free property, he can't have cancer).

**Ax. 3.**  $P(G)$

Translation: "Axiom 3. The property of being godlike is a positive property."

This is a straight-forward assumption given that a godlike object has every positive property and no negative properties.

**Th. 2.**  $\Diamond \exists x G(x)$

Translation: "Theorem 2. It is possible that there exists an object  $x$  that has the godlike property."

It is possible that God exists. A very simple line derived from combining theorem 1 and axiom 3. Because being godlike is a positive property, it is possible that that property exists in an object.

**Df. 2.**  $\varphi \text{ ess } x \Leftrightarrow \varphi(x) \wedge \forall \psi (\psi(x) \Rightarrow \Box \forall y (\varphi(y) \Rightarrow \psi(y)))$



Translation: “Definition 2. Phi is an essential property of x if and only if object x has the property phi and – for every psi, if object x has property psi then it is necessary that for every y if object y has property phi then object y has property psi.”

This line defines an essential property of an object. In order for a property to be an essential property of an object, the essential property must cause every other property that the object has. As an example, consider a piece of gourmet bread from a 5-star restaurant. Just by telling you that, you know many different properties about that item. You know that it is food, that it tastes good, that it has a soft interior and a crusty exterior, etc. Being gourmet bread is the essential property of that object because it causes all the rest of its properties.

$$\text{Ax. 4. } P(\varphi) \Rightarrow \Box P(\varphi)$$

Translation: “Axiom 4. If the property phi is a positive property then it is necessary that the property phi is a positive property.”

This line assumes that a positive property is necessarily a positive property. There’s not a whole lot going on in this line.

$$\text{Th. 3. } G(x) \Rightarrow G \text{ ess } x$$

Translation: “Theorem 3. If object x has the godlike property then the godlike property is the essential property of object x.”

This theorem establishes that “godlike-ness” is the essential property of any godlike object. An essential property is one that directly causes every other property in the object. Being godlike forces that object to have every positive property, and no negative properties. This godlike object can have no other properties besides the ones that being godlike forces it to have.

In short, all of God’s properties come directly from the fact that God is God. Why does God love well? Because he is God. Why is God the savior of the world? Because he is God. All of these other properties are true because of his core identity.

**Df. 3.**  $E(x) \Leftrightarrow \forall \varphi (\varphi \text{ ess } x \Rightarrow \Box \exists y \varphi(y))$

Translation: “Definition 3. Object x has necessary existence if and only if for every property phi, if phi is an essential property of x then it is necessary that there exists an object y that has property phi.”

This line defines existence. An object necessarily exists if its essential property also exists. There is definitely a piece of gourmet bread out there if at least one object has gourmet bread-ness.

Ax. 5.  $P(E)$

Translation: “Axiom 5: Necessary existence is a positive property.”

Existing is good!

Th. 4.  $\square \exists x G(x)$

Translation: “Theorem 4. It is necessary that there is an object  $x$  that has the godlike property.”

This is the conclusion: It is necessary that God exists.

This is the final step of the proof.

The proof for this theorem is unsurprisingly complicated, but the general flow of the proof focuses on definition 1 and axiom 5. God has every good property, and necessary existence is a good property. God must therefore exist.

As the mathematicians say, Q.E.D.

Robert J. Marks Ph.D. is Senior Fellow and Director of the Bradley Center and is Distinguished Professor of Electrical and Computer Engineering at Baylor University. Marks is a Fellow of both the *Institute of Electrical and Electronic Engineers* (IEEE) and Optica (formerly the *Optical Society of America*). He was the former Editor-in-Chief of the *IEEE Transactions on Neural Networks* and is the current Editor-in-Chief of *BIO-Complexity*.

SAMUEL HAUG

Samuel Haug is a PhD student in Electrical and Computing Engineering at Baylor University.

## POINT TO PONDER

Let  $P(X)$  be the probability of event  $X$

- 1) Let  $X$  be the event our universe was a random event  
According to Multiverse Theory there are  
 $10^{500}$  different universes

Thus  $P(X) = 10^{-500}$  a very very small number

$$10^{-500} = 0.00000 \dots 00000000000000000001$$

{ 499 (0)zeros }

- 2) Let  $X$  be the event our universe is the only universe  
and was created by Jesus or evolved from nothing  
It only has 2 possible outcomes.

Then

$$P(X) = \frac{1}{2} = 0.5 \ggg 10^{-500}$$